Key

Algebra II
Notes 5.1/5.2: nth roots and radicals
Obj: Simplify radical expressions and solve equations

Radical Expressions Vocabulary

popular cubes
1, 8, 27, 64, 125

- index $\sqrt[n]{a} = x$ radicand then $x^n = c$

- The principal root is the positive root of a number that has a positive and negative root.

Example 1. Find all the real cube roots of 125.

$x^3 = 125$
$5 \cdot 5 \cdot 5$ $x = 5$

note cube roots can be either + or − but not both
$-5 \cdot -5 \cdot -5 = -125$

Practice: Find all the real cube roots of:

1. -1000
-10
2. .008
$.2$
3. 1/27
$\frac{1}{3}$

Find all the real 4th roots of 16? $x^4 = 16$ $2 \cdot 2 \cdot 2 \cdot 2$ or $-2 \cdot -2 \cdot -2 \cdot -2$
$x = 2$ or -2

Even vs odd roots: even roots will have two solutions
ODD roots will have 1 solution

Practice: Find the real principal root.

1. $\sqrt[3]{-27}$
-3
2. $\sqrt[4]{-81}$
no solution
3. $\sqrt{25}$
$\pm 5 \rightarrow$
so 5
4. $\sqrt[3]{64}$
4

NOTE: Even roots must have positive radicands
Review: Simplifying Square roots: Look for perfect squares that are factors.

List of perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169

Simplify:

$\sqrt{24}$ $\sqrt{4}\cdot\sqrt{6}$ $2\sqrt{6}$

$\sqrt{96}$ $\sqrt{16}\cdot\sqrt{6}$ $4\sqrt{6}$

$\sqrt{48}$ $\sqrt{16}\cdot\sqrt{3}$ $4\sqrt{3}$

$\sqrt{75}$ $\sqrt{25}\cdot\sqrt{3}$ $5\sqrt{3}$

Example 2. Simplify nth roots.

A.) $\sqrt[5]{32m^5} =$

$\sqrt[5]{2^5 m^5}$

$2m$

B) $\sqrt[4]{x^{20}y^8} =$

$\sqrt[4]{x^4x^4x^4x^4x^4y^4y^4}$

$|x^5|y^2$

32

4 8

2 2 4 2

2 2 2 2 2

When to use absolute value:

If you are taking an even root and your answer has an odd power

<u>**Remove if:**</u>

even power

<u>Extra practice:</u> Simplify each radical expression

even root → $\sqrt{4x^6}$ $\sqrt{4}\ \sqrt{x^2}\cdot\sqrt{x^2}\sqrt{x^2}$

$2|x^3|$

↑

odd power

$\sqrt[3]{a^3b^6}$ $a^3b^3b^3$

ab^2

$\sqrt[4]{x^4y^8}$ $x^4y^4y^4$

$|x|y^2$

$\sqrt{4x^2y^4}$

$\sqrt{4x^2y^2y^2}$

$2|x|y^2$

$\sqrt[3]{-27c^6}$ c^3c^3

$-3c^2$

$\sqrt[4]{x^8y^{12}}$ $x^4x^4y^4y^4y^4$

$|xy^3|$

Example 3. Use nth roots to solve equations.

Solve $\frac{2x^5}{2} = \frac{64}{2}$

$x^5 = 32$

$x = \sqrt[5]{32}$

$x = 2$

You try. Solve. $\frac{5x^3}{5} = \frac{320}{5}$ and $2p^4 = 162$

$x^3 = 64$

$x = 4$

$p^4 = 81$

$p = \pm 3$

If $x^n = C$ then $x = \sqrt[n]{C}$ if n is even → 2 solutions

nth root of C

Properties of radicals:

Product Property: $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Quotient Property: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Example 4. Simplify using your properties.

a. $\sqrt[5]{16} \cdot \sqrt[5]{8}$ neither can be simplified so multiply

$\sqrt[5]{2^4 \cdot 2^3}$

$\sqrt[5]{2^7}$

$2\sqrt[5]{4}$

b. $\sqrt[3]{54x^2y^3} \cdot \sqrt[3]{5x^3y^4}$

54
9 6
$3 \cdot 3 \cdot 3 \cdot 2$
$3^3 \cdot 2$

5

$\sqrt[3]{3^3 \cdot 2 \cdot 5 x^5 y^7}$

$\sqrt[3]{3^3 \cdot 2 \cdot 5 \cdot x^3 \cdot x^2 y^3 \cdot y^3 \cdot y}$

$3xy^2 \sqrt[3]{10x^2y}$

Extra Practice: Multiply and simplify answer if necessary.

$\sqrt{3}\cdot\sqrt{12} = \sqrt{36}$

$= 6$

$\sqrt[3]{3}\cdot\sqrt[3]{-9}$ $\sqrt[3]{-27} = -3$

40
$\wedge$
4 10
2·2·2 5

$\sqrt[3]{40n^2}\cdot\sqrt[3]{2n^3}$

$\sqrt[3]{2^3\cdot 5\cdot 2n^5} = 2n\sqrt[3]{10n^2}$

$\sqrt[3]{2^3\cdot 5\cdot 2n^3 n^2}$

$\sqrt{5x^2}\cdot\sqrt{24x^5}$

$\sqrt{5\cdot x^7\cdot 2^3\cdot 3}$

$\sqrt{5\cdot x^2\cdot x^2\cdot x^2\, x\; 2^2\cdot 2\cdot 3}$

$2|x^3|\sqrt{30x}$

24
$\wedge$
6 4
2 3 2 2

Day 2.

Example 5. Rationalize a denominator.

a. $\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

↑make bottom a perfect square

Divide First

b. $\frac{\sqrt{x^3}}{\sqrt{5xy}} = \frac{\sqrt{x^2}}{\sqrt{5y}} = \frac{x}{\sqrt{5y}}\cdot\frac{\sqrt{5y}}{\sqrt{5y}}$

$= \frac{x\sqrt{5y}}{5y}$

c. $\sqrt{\frac{2n}{9m}} = \frac{\sqrt{2n}}{3\sqrt{m}}\cdot\frac{\sqrt{m}}{\sqrt{m}}$

$= \frac{\sqrt{2nm}}{3m}$

You try. Assume all variables are positive.

$\sqrt{\frac{7}{16x^3}}$ $\frac{\sqrt{7}}{4x\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$

$\frac{\sqrt{7x}}{4x^2}$

$\sqrt{\frac{2}{3x}}\cdot\frac{\sqrt{3x}}{\sqrt{3x}}$

$\frac{\sqrt{6x}}{3x}$

$\sqrt{\frac{7}{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35}}{5}$

Example 6: Adding and subtracting radicals.

Only: must be same under $\sqrt{\ }$

$2\sqrt{3}+4\sqrt{3} = 6\sqrt{3}$

$2\sqrt{3} + 3\sqrt{5} \neq$ can't add

not same

simplify first to $5\sqrt{3}$

Add or subtract if possible. It may be necessary to simplify the radicals first to see if you have like terms.:

$\sqrt{27}=3\sqrt{3}$

$12\sqrt{4\cdot 3}$

$5\sqrt[3]{x}-3\sqrt[3]{x}$

$2\sqrt[3]{x}$

$75 \to 25 \cdot 3$

$5\sqrt{75}+2\sqrt{12}$

$5\cdot 5\sqrt{3}+2\cdot 2\sqrt{3}$

$25\sqrt{3}+4\sqrt{3}$

$29\sqrt{3}$

$2\sqrt{3}+3\sqrt{27}$

$2\sqrt{3}+3\cdot 3\sqrt{3}$

$2\sqrt{3}+9\sqrt{3}$

$11\sqrt{3}$

$\sqrt{50}=5\sqrt{2}$

$\sqrt{32}=4\sqrt{2}$

$\sqrt{12}=2\sqrt{3}$

$\sqrt{50}-4\sqrt{32}+3\sqrt{12}$

$5\sqrt{2}-4\cdot 4\sqrt{2}+2\cdot 3\sqrt{3}$

$5\sqrt{2}-16\sqrt{2}+6\sqrt{3}=\boxed{-11\sqrt{2}+6\sqrt{3}}$

$\sqrt{20}-\sqrt[3]{16}+\sqrt[3]{250}-\sqrt{5}$

$2\sqrt{5}-2\sqrt[3]{2}+5\sqrt[3]{2}-\sqrt{5}$

$\boxed{\sqrt{5}+3\sqrt[3]{2}}$

$\sqrt{20}=2\sqrt{5}$

$\sqrt[3]{16}=2\sqrt[3]{2}$

$\sqrt[3]{250}=5\sqrt[3]{2}$

$250 \to 25 \cdot 10 \to 5\cdot 5\cdot 5\cdot 2$

Example 7. Multiplying Radical Expressions

A) $\sqrt[3]{7}(2-\sqrt[3]{49})$ $7\cdot 7^2$

$2\sqrt[3]{7}-\sqrt[3]{147}$ $147 \to 7\cdot 7\cdot 7$

$2\sqrt[3]{7}-7$

B.) $(2x-\sqrt{3})(2x-\sqrt{3})$ FOIL

$4x^2-2x\sqrt{3}-2x\sqrt{3}+\sqrt{9}$

$4x^2-4x\sqrt{3}+3$

You try. $(3+2\sqrt{5})(2+4\sqrt{5})$

$6+12\sqrt{5}+4\sqrt{5}+8\sqrt{25}$

$6+16\sqrt{5}+8\cdot 5$

$\boxed{46+16\sqrt{5}}$

$(-2+2\sqrt{5})(6-2\sqrt{5})$

$-12+4\sqrt{5}+12\sqrt{5}-4\sqrt{25}$

$-12+16\sqrt{5}-4\cdot 5$

$\boxed{-32+16\sqrt{5}}$

$\sqrt{8}=2\sqrt{2}$

$(7-6\sqrt{8})^2=(7-12\sqrt{2})(7-12\sqrt{2})$

$6\cdot 2\sqrt{2}$

$7-12\sqrt{2}$

$49-84\sqrt{2}-84\sqrt{2}+144\cdot\sqrt{4}$

$49-168\sqrt{2}+144\cdot 2=\boxed{337-168\sqrt{2}}$

Multiplying Conjugates:

$(\sqrt{5}+\sqrt{6})(\sqrt{5}-\sqrt{6})$

$\sqrt{25}-\sqrt{30}+\sqrt{30}-\sqrt{36}$

$5-6$

-1

no more $\sqrt{\ }$

Example 8. Rationalizing Binomial Denominators

- Multiply by a fraction of $\frac{\text{conjugate of denominator}}{\text{conjugate of denominator}}$. Simplify.

$$\frac{1}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{\underbrace{4-2\sqrt{5}+2\sqrt{5}-\sqrt{25}}_{4-5}} = \frac{2-\sqrt{5}}{-1} = -2+\sqrt{5}$$

$\sqrt{25} \downarrow 5$

$$\frac{5-\sqrt{2}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{10+5\sqrt{3}-2\sqrt{2}-\sqrt{6}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}}$$

$$\frac{10+5\sqrt{3}-2\sqrt{2}-\sqrt{6}}{1}$$

$$\frac{-4x}{1-\sqrt{x}} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{-4x-4x\sqrt{x}}{1-\sqrt{x}+\sqrt{x}-\sqrt{x^2}}$$

$$= \frac{-4x-4x\sqrt{x}}{1-|x|}$$